In the expression we are supposed to integrate, the cos() function has an "inner" function (x^2) and it is multiplied by an "outer" differential expression ($2x\,\mathrm{d}x$). In order to do this kind of integral, you have to be able to recognize on sight that the outer differential expression is equal to the differential of the inner function.

In other words, you have to be able to see $\int \cos(x^2)\,2x\,\mathrm{d}x$ and say, "Hey! The inside and outside of that cosine function are related! I wonder if I set u to the inner function, will $\mathrm{d}u$ wind up being the outside function (or something close to it)?"

So, then, you set u to be the inside of the cosine function (i.e., x^2), and see what $\mathrm{d}u$ becomes. Well, if $u = x^2$ then $\mathrm{d}u = 2x\,\mathrm{d}x$, which is exactly what the outer function is. If we do both of these substitutions, we get:

$$\int \cos(u)\,\mathrm{d}u$$

That's easy to integrate! This just becomes $\sin(u)$. Of course, after integrating, we need to back-substitute in for u, which will give us $\sin(x^2)$.

In your mind, when you see something like $\cos(x^2)\,2x\,dx$, the expression $2x\,\mathrm{d}x$ should just jump out at you as being the differential of x^2. If you have practiced your differentials enough, your mind should pick this up automatically. If it doesn't, I would suggest going back and reworking problems from Part II, and copying the table in Appendix H.6 several times.

Never forget that you must replace *both* the expression for x as well as the expression for $\mathrm{d}x$! If you wind up with a mix of xs and us at the end then you have probably done it wrong. You have to be able to substitute for both the xs and the $\mathrm{d}x$s or it doesn't work.

— **Example 19.1**

Find $\int e^{x^3}\,3x^2\,\mathrm{d}x$.

Hopefully when you looked at the problem it immediately popped out at you that $3x^2\,\mathrm{d}x$ is the differential of x^3. Therefore, we can set $u = x^3$ and therefore $\mathrm{d}u = 3x^2\,\mathrm{d}x$. This can be substituted back into the equation to become:

$$\int e^u\,\mathrm{d}u$$

This is a super-easy integral:

$$\int e^u\,\mathrm{d}u = e^u + C$$

We are almost done—we just need to back-substitute in for u. $u = x^3$, therefore, this becomes:

$$e^{x^3} + C$$

Putting it all together, we have:

$$\int e^{x^3}\,3x^2\,\mathrm{d}x = e^{x^3} + C$$

If you are worried about any of the steps, just go back and differentiate your results to see if you get the original expression back.