4. **Question:** Find $\lim_{x\to\infty} \frac{32x^6-65x^4+33}{88x^5+23x^4-25x^3+22x^2+100}$

   Solution: ∞

   Explanation: When dealing in limits to infinity, only the highest degree term from the numerator and the highest degree term from the denominator affect the outcome. In the numerator, the highest degree term is $32x^6$. In the denominator, the highest degree term is $88x^5$.

   Therefore, the problem reduces to:

$$\frac{32x^6}{88x^5}$$

   Canceling the xs gives us:

$$\frac{32x}{88}$$

   Notice that there is an x left in the numerator. Therefore, as x travels towards infinity, this value will become infinitely large. Since we are traveling to positive infinity, the result will be a positive infinity.

   This can also be seen using hyperreals. If we substitute ω for x, we get $\frac{32}{88}\omega$, which is a hyperreal infinity. The standard part is ∞.

5. **Question:** Find $\int_5^\infty \frac{7}{x^3}\,\mathrm{d}x$

   Solution: $\frac{21}{625}$

   Explanation: To find an integral with infinite limits of integration, first perform the indefinite integral normally, and then use limits to determine any results at infinity.

$$\begin{aligned}\int_5^\infty \frac{7}{x^3}\,\mathrm{d}x &= \int \frac{7}{x^3}\,\mathrm{d}x\Big|_5^\infty \\ &= \int 7x^{-3}\,\mathrm{d}x\Big|_5^\infty \\ &= -21x^{-4}\Big|_5^\infty \\ &= -21(\infty)^{-4} - -21(5)^{-4} \\ &= \frac{-21}{\infty^4} + \frac{21}{625}\end{aligned}$$

   Here, the term $\frac{-21}{\infty^{-4}}$ can be evaluated using limit techniques. Since the infinity was in the denominator, $\frac{-21}{\infty^4}$ reduced to zero. Therefore, we can complete this evaluation as follows:

$$\begin{aligned}\lim_{h\to\infty} \frac{-21}{h^4} + \frac{21}{625} &= 0 + \frac{21}{625} \\ &= \frac{21}{625}\end{aligned}$$

6. **Question:** Find $\int_0^\infty x^2\,\mathrm{d}x$

   Solution: ∞

   Explanation: To find an integral with infinite limits of integration, first perform the indefinite integral normally, and then use limits to determine any results at infinity.

$$\begin{aligned}\int_0^\infty x^2\,dx &= \int x^2\,\mathrm{d}x\Big|_0^\infty \\ &= \frac{x^3}{3}\Big|_0^\infty \\ &= \frac{(\infty)^3}{3} - \frac{(0)^3}{3} \\ &= \frac{(\infty)^3}{3} - 0 \\ &= \frac{(\infty)^3}{3}\end{aligned}$$

   The ∞ tells us we need to use our limit rules to determine the answer. Since the ∞ is in the numerator, the term $\frac{\infty^3}{3}$ will grow infinitely large. Therefore, it iself will become an infinite term.

   Therefore, the answer is ∞.

7. **Question:** Find $\int_{-\infty}^{-3} 2x^{-5}\,\mathrm{d}x$

   Solution: $-\frac{1}{162}$

   Explanation: To find an integral with infinite limits of integration, first perform the indefinite integral normally, and then use limits to determine any results at infinity.

$$\begin{aligned}\int_{-\infty}^{-3} 2x^{-5}\,\mathrm{d}x &= \int 2x^{-5}\,\mathrm{d}x\Big|_{-\infty}^{-3}\\ &= \frac{2x^{-4}}{-4}\Big|_{-\infty}^{-3}\\ &= -\frac{1}{2x^4}\Big|_{-\infty}^{-3}\\ &= -\frac{1}{2(-3)^4} - -\frac{1}{2(-\infty)^4}\\ &= -\frac{1}{162} + \frac{1}{2(-\infty)^4}\end{aligned}$$

The presence of ∞ tells us that we need to use our limit rules to determine the answer. Since the infinity is in the denominator, the term $\frac{1}{2(-\infty)^4}$ will get infinitely close to zero. In other words:

$$\lim_{h\to-\infty} \frac{1}{2h^4} = 0$$

This reduces the expression to:

$$-\frac{1}{162} + 0$$

Or just $-\frac{1}{162}$.

8. **Question:** Find $\lim_{x\to\infty} \frac{2x^3-5x^9+3x^2+500}{3x^5-2x^4+12x^2+8x^9-5000}$

   Solution: $-\frac{5}{8}$

   Explanation: When dealing in limits to infinity, only the highest degree term from the numerator and the highest degree term from the denominator affect the outcome. In this case, the highest degree is not in the first position—you have to actually look at the problem to find it.

   On the numerator, the highest degree term is $-5x^9$. On the denominator it is $8x^9$. Therefore, as x approaches infinity, the other terms become infinitely less important. Therefore, this fraction reduces to:

   $$\lim_{x\to\infty} \frac{-5x^9}{8x^9}$$

   The xs cancel out, which just leaves $-\frac{5}{8}$

9. **Question:** Find $\lim_{x\to-\infty} \frac{x^4+x^5}{x^3-5x^4}$

   Solution: ∞

   Explanation: When dealing in limits to infinity, only the highest degree term from the numerator and the highest degree term from the denominator affect the outcome. Therefore, this fraction reduces to $\frac{x^5}{-5x^4}$. This reduces to $-\frac{x}{5}$. As x approaches $-\infty$, this will get bigger and bigger (because the negatives are cancelling each other out). Therefore, the result will be ∞.

10. **Question:** Find $\lim_{x\to\infty} \frac{\sin(x)}{x}$

    Solution: 0

    Explanation: This limit is interesting because the numerator is oscillating. Think about the sine function—it goes back and forth *forever* between -1 and 1. It does not have a single stable value at all. However, the numerator has a stable *range*—it never goes above 1 or below -1. This will allow us to reason about it better.

    Now thing about the denominator. As x goes to infinity, the denominator is getting bigger and bigger. Note that the numerator is *not* getting bigger and bigger. It isn't just sitting there, but it is always bounded within its box, from -1 to 1.

    Therefore, since dividing *any* value in that range by ∞ has a limit of 0, that means that this function must have a limit of 0. Even though we don't know what *specific* value to assign for $\sin(x)$, we know that no matter which value in its range we were to assign it, the limit would be 0.