everything we need to solve for $\frac{\mathrm{d}\theta}{\mathrm{d}t}$:

$$\begin{aligned}
\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \frac{-1}{L\sin(\theta)}\frac{\mathrm{d}B}{\mathrm{d}t} \\
\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \frac{-1}{11\,\mathrm{ft}\,\sin(1.1986)} \cdot 2\,\frac{\mathrm{ft}}{\mathrm{sec}} \\
\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \frac{-2}{11\,\sin(1.1986)\,\mathrm{sec}} \\
\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \frac{-2}{10.2465\,\mathrm{sec}} \\
\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \frac{-0.1952}{\mathrm{sec}}
\end{aligned}$$

Note that there was no resulting unit for the angle, because radian measurements are often treated as having either no dimensions or an implicitly assumed dimension. It is usually read as "−0.1952 radians per second," but it is only written as $\frac{-0.1952}{\mathrm{sec}}$. It is not wrong, however, if you wrote $\frac{-0.1952\,\mathrm{radians}}{\mathrm{sec}}$.

8. **Question:** A pebble is dropped into a pond, generating circular ripples. The radius of the largest ripple is increasing at a constant rate of 2 inches per second. After 5 seconds, what is the rate that the *circumference* of the largest ripple is increasing?

Solution: The rate that the circumference of the largest ripple is increasing is $12.5664\,\frac{\mathrm{in}}{\mathrm{sec}}$

Explanation: This question explores the relationship between the radius and the circumference. This is given by the equation:

$$D = 2\pi R$$

To convert this into a related rate equation, we take the differential and then divide by $\mathrm{d}t$:

$$\begin{aligned}
D &= 2\pi R \\
\mathrm{d}D &= 2\pi \mathrm{d}R \\
\frac{\mathrm{d}D}{\mathrm{d}t} &= 2\pi\,\frac{\mathrm{d}R}{\mathrm{d}t}
\end{aligned}$$

The problem states that the radius is increasing at a rate of $2\,\frac{\mathrm{in}}{\mathrm{sec}}$, which is $\frac{\mathrm{d}R}{\mathrm{d}t}$. Note that this equation *doesn't have* R itself in the equation, so we don't need to solve for it, even though we could (therefore, the answer will be the same no matter how many seconds we wait). Therefore, we can substitute this rate back into the related rate equation and get:

$$\begin{aligned}
\frac{\mathrm{d}D}{\mathrm{d}t} &= 2\pi\,\frac{\mathrm{d}R}{\mathrm{d}t} \\
\frac{\mathrm{d}D}{\mathrm{d}t} &= 2\pi\, 2\,\frac{\mathrm{in}}{\mathrm{sec}} \\
\frac{\mathrm{d}D}{\mathrm{d}t} &= 4\pi\,\frac{\mathrm{in}}{\mathrm{sec}} \\
\frac{\mathrm{d}D}{\mathrm{d}t} &\approx 12.5664\,\frac{\mathrm{in}}{\mathrm{sec}}
\end{aligned}$$

9. **Question:** A hose is inflating a spherical baloon which begins empty. The water is coming out of the hose at a rate of $4\frac{\mathrm{mL}}{\mathrm{sec}}$ ($1\mathrm{mL} = 1\mathrm{cm}^3$). After 10 seconds, what is the rate of the radius of the baloon increasing?

Solution: $0.0707\frac{\mathrm{cm}}{\mathrm{sec}}$

Explanation: In solving this problem, we will use millimeters for length units to match the volume units.

So, the abstract geometry of the equation is that of a sphere.

The equation that relates the radius (r) to the volume (v) of a sphere is:

$$v = \frac{4}{3}\pi r^3$$

To convert this to a related rate equation, we simply take the differential and divide by $\mathrm{d}t$:

$$\begin{aligned}
v &= \frac{4}{3}\pi r^3 \\
\mathrm{d}v &= 3\frac{4}{3}\pi r^2\,\mathrm{d}r \\
\mathrm{d}v &= 4\pi r^2\,\mathrm{d}r \\
\frac{\mathrm{d}v}{\mathrm{d}t} &= 4\pi r^2\,\frac{\mathrm{d}r}{\mathrm{d}t}
\end{aligned}$$

We want to know the rate that the radius is increasing, which is $\frac{\mathrm{d}r}{\mathrm{d}t}$. So we can just solve for that:

$$\begin{aligned}
\frac{\mathrm{d}v}{\mathrm{d}t} &= 4\pi r^2\,\frac{\mathrm{d}r}{\mathrm{d}t} && \text{related rate equation} \\
\frac{1}{4\pi r^2}\frac{\mathrm{d}v}{\mathrm{d}t} &= \frac{\mathrm{d}r}{\mathrm{d}t} && \text{solved for } \frac{\mathrm{d}r}{\mathrm{d}t}
\end{aligned}$$

So, the pieces that we need to solve are the radius (r) and the rate of change in volume ($\frac{\mathrm{d}r}{\mathrm{d}t}$). The water coming through the hose is a change in *volume*. Therefore, the rate that the water coming through the hose is the rate that the volume of the sphere is increasing, and therefore represents $\frac{\mathrm{d}v}{\mathrm{d}t}$. This is given by the problem as $4\frac{\mathrm{mL}}{\mathrm{sec}}$.

The second thing we need is the radius. The radius starts at 0cm. However, we need to know what the radius will be *after* 10 *seconds*. Because the flow rate of the water is constant, we can easily calculate the *volume* after ten seconds:

$$4\frac{\mathrm{mL}}{\mathrm{sec}} \cdot 10\mathrm{sec} = 40\mathrm{mL}$$

We can then use the equation for the volume of a sphere to solve for r:

$$\begin{aligned}
\frac{4}{3}\pi r^3 &= v \\
\frac{4}{3}\pi r^3 &= 40\mathrm{mL} \\
r^3 &= \frac{40\mathrm{mL}}{\frac{4}{3}\pi} \\
r^3 &= \frac{3 \cdot 40\mathrm{mL}}{4\pi} \\
r^3 &= \frac{30\mathrm{mL}}{\pi} \\
r &= \sqrt[3]{\frac{30\mathrm{mL}}{\pi}} \\
r &\approx 2.1216\mathrm{cm}
\end{aligned}$$

Note in that last step, because $1\mathrm{mL} = 1\mathrm{cm}^3$, when we took the square root it switched from mL to cm because $\sqrt[3]{1\mathrm{mL}} = \sqrt[3]{1\mathrm{cm}^3} = 1\mathrm{cm}$.

Now we have all of the pieces to substitute into our related rate equation:

$$\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}t} &= \frac{1}{4\pi r^2}\frac{\mathrm{d}v}{\mathrm{d}t} \\
\frac{\mathrm{d}r}{\mathrm{d}t} &= \frac{1}{4\pi(2.1216\mathrm{cm})^2}\left(4\frac{\mathrm{mL}}{\mathrm{sec}}\right) \\
\frac{\mathrm{d}r}{\mathrm{d}t} &= \frac{1}{\pi(2.1216\mathrm{cm})^2}\frac{\mathrm{mL}}{\mathrm{sec}} \\
\frac{\mathrm{d}r}{\mathrm{d}t} &\approx \frac{1}{\pi 4.5012\mathrm{cm}^2}\frac{\mathrm{mL}}{\mathrm{sec}} \\
\frac{\mathrm{d}r}{\mathrm{d}t} &\approx \frac{1}{14.1410\mathrm{cm}^2}\frac{\mathrm{mL}}{\mathrm{sec}} \\
\frac{\mathrm{d}r}{\mathrm{d}t} &\approx 0.0707\frac{\mathrm{mL}}{\mathrm{cm}^2\,\mathrm{sec}} \\
\frac{\mathrm{d}r}{\mathrm{d}t} &\approx 0.0707\frac{\mathrm{cm}^3}{\mathrm{cm}^2\,\mathrm{sec}} \\
\frac{\mathrm{d}r}{\mathrm{d}t} &\approx 0.0707\frac{\mathrm{cm}}{\mathrm{sec}}
\end{aligned}$$

Therefore, after ten seconds, the radius of the baloon is increasing at a rate of $0.0707\frac{\mathrm{cm}}{\mathrm{sec}}$.

10. **Question:** A plane is flying at a constant altitude of 3 miles down a straight road at 150 miles per hour with a radar gun to catch speeding vehicles. The radar gun, however, only determines the speed of the vehicle *with relation to the airplane* (i.e., how fast the distance between the plane and the vehicle is increasing or decreasing) and the distance to the vehicle. If a vehicle headed *the same direction of the plane* is clocked by the radar gun as coming toward you at 80 miles per hour at a distance of 5 miles, how fast is the vehicle going?

Solution: $50\frac{\mathrm{mi}}{\mathrm{hr}}$

Explanation: If you think about the relationship between the plane, the ground, and the vehicle, you recognize that this is a right triangle. We will call the altitude of the plane A, the distance along the ground as G, and the direct distance through the air between the plane and the vehicle as D. Therefore, we know that the following relationship exists among these variables:

$$A^2 + G^2 = D^2$$

The altitude is given—3mi. The air distance is given by the radar gun—5mi. Therefore, we can